THE UNIFIED FRACEP POTENTIAL:
For Positive and Negative Mass Sources at All Scales
Part c: the fundamental scale (update 10/28/19)

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ABSTRACT: The Standard Model (SM) faces challenges in its attempts to explain dark matter. FRACEP offers an alternative view – composite elementary particles with both positive and negative mass components. It uses a single function to characterize the behavior of its particle-interactions at all scales. For positive mass sources, it agrees with current data-consistent models that are positive-mass based. This paper examines the behavior of FRACEP at the scale of its fundamental and its composite elementary particles. It hypothesizes that the negative mass potential at this scale may indicate a possible explanation for the Big Bang inflation energy.

INTRODUCTION: The Lao Tse (Tao-Te Ching) tells the Chinese wisdom about the nature of the Universe: the "being" and the "non-being" where "these two spring from the same source" – "the Darkness within the Darkness, the gate to all mystery" [1]. Although Lao Tzu's words are ancient, they sound strangely reminiscent of the modern puzzle of dark matter and dark energy and their relation to the "Bright Universe" we see (the "being"). They hint of a "Dark Universe" we cannot see (the "non-being" – the unseen dark matter and energy within the darkness of the cosmos).

The dark (unseen) matter (DM) hypothesis made its debut in the late 1930's because of an apparent insufficiency in the observed mass in the cosmic motions [2]. The nature of this matter is still unclear. Chang proposes that negative mass is a possible answer to DM, and, he has developed field equations describing the repulsive interaction between positive and negative mass [3]. Evidence for accelerating expansion in the universe, in 1997 [4], led to the assumption of dark energy (DE), whose nature is also a matter of debate [5].

The predominating consensus of cosmologists, today, is the DM and DE paradigm [6]. The contents of the universe by this paradigm are divided as: DE (identity unknown), ~73%; DM (identity unknown), ~23%; other non-luminous matter (gasses, neutrinos and super massive black holes) and luminous matter (stars, gasses and radiation), ~4%. Numerous possible candidates have been proposed for DM and DE – many focusing on the large scale [7] [8] [9]. There is also, however, an interest in the possibility of DM at the small scale.

Feng [10] [11] notes that there are good reasons to consider the Fermi scale as a possibility for DM particles because this regime is directly testable and would contribute to the understanding of the electroweak symmetry breaking. Such particles often automatically have the right properties for DM, and, simulations show they are naturally produced with the required cosmological densities. Kile and Soni [12] considered the possibility of observing light fermionic DM particles based on their inelastic interaction leading to the annihilation into a particle with smaller mass. Bromm et al. [13] used simulations to explore the small-scale fluctuation in cold DM. They demonstrated the recreation of the typical structure of the gas that contributes to primordial star formation.

Finally, the ALPHA collaboration at CERN [14] is considering from a different perspective. Big Bang theory indicates that, during the creation event, equal amounts of matter and anti-matter should have been created. They, in turn, should have annihilated each other; but, an excess of matter has survived, indicating that the laws of nature, as we understand them, do not apply equally to matter and anti-matter. The ALPHA collaboration is experimentally exploring that puzzle. Specifically, assuming the hydrogen atom is purely positive mass, is the anti-hydrogen atom purely positive (with opposite characteristics), or is it partly negative mass causing it to violate the expected gravitational attraction because there is a difference in the behavior of matter and anti-matter? Their initial efforts [15] created sufficiently stable anti-hydrogen, and further experiments are planned to more completely answer the question.

In a similar vein, the FRACEP model [16] develops composite versions of the Standard Model (SM) elementary fermions and bosons, having components that can have positive, as well as, negative mass. This model leads to an additional set of negative-mass particles that have the potential to provide some options in the DM search, and, the repulsion between opposite mass types that may contribute to the DE identification. To address the interaction of these positive and negative mass particles, a unified potential is under development. Two earlier works [17] [18] focused on the potential's behavior, at larger scales for both positive and negative mass sources. This paper addresses the potential's behavior at the smallest scales (the FRACEP fundamental particles ($\mathcal{G}p$ and $\mathcal{G}p$), and the composite elementary particles.

THE FRACEP POTENTIAL: The FRACEP potential (V_{FRACEP}) is a unified potential that characterizes both positive and negative mass sources at quantum scales through the largest cosmic scales with a single multi-term function. It is consistent with currently accepted observation driven models (based on positive mass only – the assumed type). In its most general form, it is a function of both time and space, that is, $V_{FRACEP}(r,t) = F(t) \cdot V(r)$. The $F(t)$ describes the temporal behavior that is important to the decay times of the composite fermions, as well as, the oscillation times associated with the charge and spin characteristics of the charge and spin carriers. However, this work considers only the spatial behavior (i.e., $V_{FRACEP} = V(r)$). The time-dependent part, $F(t)$, will be considered in a later work. The general functional form for the spatial part is:

$$V_{FRACEP} = m_T \cdot \{A_0(1/M) + B_0(\sqrt{M})\} \cdot E_0(r, M) \cdot \sin\{S(r, M) + T(r, \sqrt{M})\}.$$

The FRACEP model is based on two fundamental bosons, $\mathcal{G}p$ and $\mathcal{G}n$, (positive and negative mass respectively). It proposes composite versions of the SM elementary particles (e.g. fermions, bosons, anti-fermions and anti-bosons) – the FRACEP "Bright Universe" – containing mostly positive mass with some negative mass components. In addition, there is a parallel set of particles (the FRACEP "Dark Universe") containing mostly negative mass with some positive mass components [16]. Particles having both positive mass and negative mass components are unstable, and associated with a decay half-life. V_{FRACEP} was developed to characterize the interaction of these particles.

THE MATHEMATICAL FORM OF V_{FRACEP}: The mathematical terms of V_{FRACEP} are:

(1a) $$V_{FRACEP} = m_T \cdot (A_0(M) + B_0(\sqrt{M})) \cdot \sin\{S(r,M) + T(r, \sqrt{M})\} \cdot E_0(r,M).$$

(1b) $$A_0 = 1 / (0.18 \cdot M) \; ; \; B_0 = 9.2095\text{x}10^{-8} \cdot \sqrt{M} \; ;$$
$$E_0 = \exp(-2.4 \cdot r / |M + Mf|) \; ; \; Mf = (m_{\mathcal{G}p} / m_\pi)/M.$$

(1c) $$S(r,M) = K_1 + K_f \; ;$$
$$K_1 = -0.09 \cdot 150 \cdot (\pi/180) \cdot (r /M)^2 \cdot E_1 \; ; \; E_1 = \exp(-67 \cdot |Mf|)$$
$$K_f = -0.000092 \cdot (\pi/180) \cdot [M / 8\text{x}10^{60}]^2 \cdot [1.496\text{x}10^{26}]^3.$$

(1d) $$T(r, \sqrt{M}) = K_2 + K_3 + K_4$$
$$K_2 = (-0.00006/m_\pi) \cdot 150 \cdot (\pi/180) \cdot \sqrt{M} / r \; ;$$
$$K_3 = 150 \cdot (\pi/180) \cdot E_1 /\sqrt{M} \; ; \; K_4 = K3 / r.$$

For negative mass sources ($M < 0$), the sine function in (1a) is expressed as:

(2) $$\sin(S + i\,T) = \{\sin(S) \cdot \cosh(T)\} + \{\cos(S) \cdot i \sinh(T)\},$$

which gives:

(3) $$V_{FRACEP}\,(M<0) = m_T \cdot E_0(r,M) \cdot \Big\{ [A_0 \cdot \sin(S) \cdot \cosh(T) - B_0 \cdot \cos(S) \cdot \sinh(T)] + i\,[A_0 \cdot \cos(S) \cdot \sinh(T) + B_0 \cdot \sin(S) \cdot \cosh(T)] \Big\}$$

The result of this change is not a simple redistribution of the value of the Newtonian potential (V_{Newton}) between the real and imaginary parts of V_{FRACEP}.

The $M = m_s/ m_\pi$ where m_s is the source mass for which the potential is computed. The $m_\pi = 139.57$ MeV/c^2 (the mass of the pi meson), and $m_{\mathcal{G}p} = 1.72\text{x}10^{-22}$ MeV/c^2 (the mass of the positive FRACEP fundamental particle). The m_T is the responding test mass, and, π is the constant 3.14159. All masses are in MeV/c^2, r is in fermis, and V_{FRACEP} is in MeV [19]. It is assumed that an absolute value on M and Mf in E_0 (1b) and E_1 (1c) is needed to guarantees that the exponentials (and the potential) decays at large distances for negative source mass.

THE V_{FRACEP} FUNDAMENTAL SCALE BEHAVIOR: The fundamental scale of interest here centers about the FRACEP fundamental particles (*Gp* and *Gn*, mass = $\pm 1.72\text{x}10^{-22}$ MeV/c^2, size $r_0 = 3.3\text{x}10^{-20}$ fm), and the Bright Universe composite particles and their Dark Universe counterparts (such as, the up-quark, mass = ± 3.568 MeV/c^2, size r_0 = ~$1.4\text{x}10^{-5}$ fm, and, the electron, mass = ± 0.5 MeV/c^2, size $r_0 = 4\text{x}10^{-6}$ fm).

The SM pi-meson (mass = 139.57 MeV/c^2, size r_0 ~$1.3\text{x}10^{-4}$ fm) was used to define the quantum scale behavior of V_{FRACEP}. The initial function was then modified to extend the potential down to the fundamental scale. This modification was implemented as a change in E_0 from the original factor ($E_0 = \exp[-2.4r/M]$) to the E_0 shown in (1b). A second change was the addition of E_1 (in (1c)). Both changes affect only source masses less than the pi-meson. The potentials for the pi-meson through the largest cosmic source masses remain unchanged.

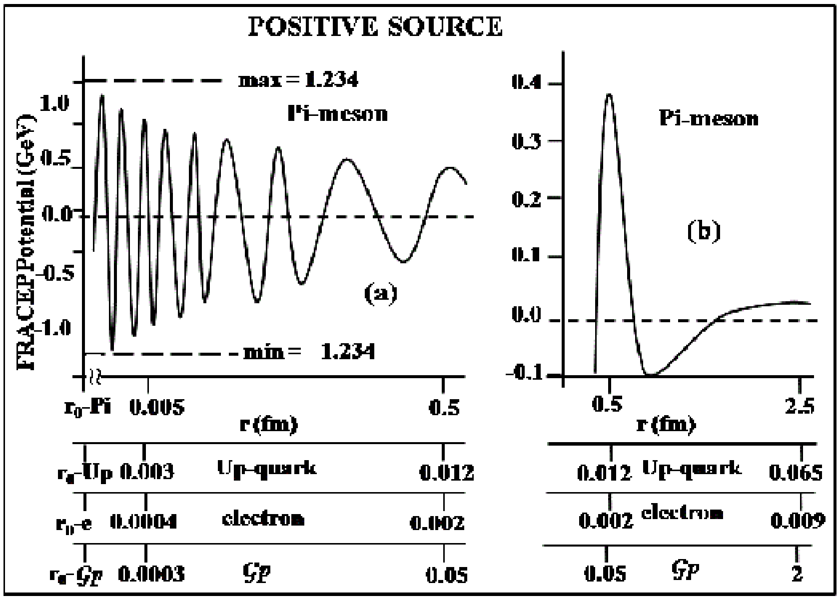

Figure 1: This shows the typical behavior of V_{FRACEP} for positive sources at the quantum through the fundamental scales. The plot shows the potential for the pi-meson (the top scale). The bottom scales show where the up-quark, the electron and the fundamental particle, *Gp* have approximately the same values as the pi-meson. In each case, the test mass (m_t) equals the source mass (m_s).

Figure 1 shows the typical behavior of V_{FRACEP} for a positive source at the quantum scale as small as its fundamental particle mass. Oscillation begins at the smallest possible *r*-value (within a few radii of the given particle) with an amplitude maximum of 1.234 GeV. The potential's amplitude begins a slow oscillating decay, eventually making a sharp step-like drop to then continue its oscillating decay to zero. For example, V_{FRACEP}

reaches as low as ~10^{-15} GeV for the: pi-meson at r ~17.5 fm; up-quark at r ~ 0.81 fm; electron at r ~0.325; and $\mathcal{G}p$ at r ~ 15 fm – after which it continues to decline at larger r.

For comparison, the figure shows V_{FRACEP} for the pi-meson (m-source = m_s = 139.57 MeV/c2, and m-test = m_t = m_s) for near r (a) and for the SM valid range (b). Three scales below, in both (a) and (b), are for the r-values where the potentials of the other particles (the up-quark, the electron, and $\mathcal{G}p$) are roughly equivalent to the pi-meson potential values. The potentials in those cases were computed for $m_s = m_t$ for the given particles. In each case, the range of r-values, and the point where the dramatic oscillation decay occurs, changes as the mass changes – though not as orderly as one might hope.

Figures 2 and 3 show a similar comparison for the same set of particles for the negative mass sources. Figure 2 compares the real and imaginary parts of the negative pi-meson potential with the real and imaginary parts of the potential for the negative fundamental particle, $\mathcal{G}n = -\mathcal{G}p$ (the two scales below). Here, the oscillating decay following the sharp step-like drop begins at the far right of the scale on (a). Over the whole range, the real and imaginary parts of the potential are slightly out of phase because of the oscillation frequency difference between the two.

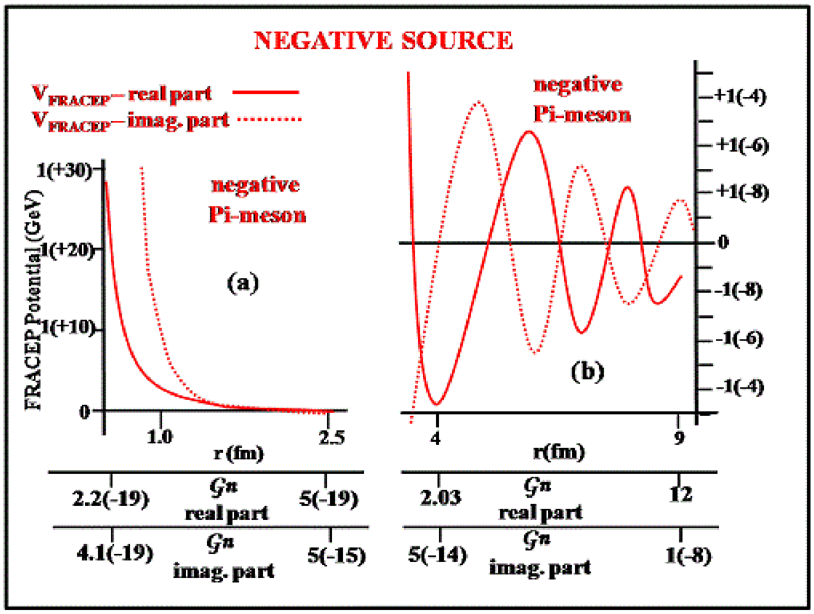

Figure 2: This shows the typical behavior of V_{FRACEP} for negative sources at the quantum through the fundamental scales. The plot shows the potential for the pi-meson (the top scale). The bottom two scales show where the real part and the imaginary part of the $\mathcal{G}n$ have approximately the same values as the pi-meson. In each case, m_s is negative, and the m_t is positive and equal to the negative of m_s (that is positive mass). For example, for the fundamental particle, $m_s = \mathcal{G}n$ and $m_t = -\mathcal{G}n = \mathcal{G}p$.

The gap between (a) and (b) (a continuing oscillating decay) is much larger for the real part of the $\mathcal{G}n$ potential than for its imaginary part or for the negative pi-meson. Also, the total range of the oscillating decay for both real and imaginary parts of $\mathcal{G}n$ is much larger than for the negative pi-meson.

For this negative mass case, both the real and imaginary parts of the negative pi-meson potential decay to ~10^{-15} GeV at about the same rate as its positive mass potential did, and to about the same *r*-value. However, the real and imaginary parts of the $\mathcal{G}n$ potential decay at vastly different rates from each other. The real part reaches ~10^{-15} GeV at r ~ 15 fm (like its positive mass potential), while for the imaginary part, r ~ 0.5 fm. The cause for this difference requires further investigation.

Figure 3 compares the negative electron with the negative up-quark. The negative pi-meson potential over the range has already transitioned to what appears to be a non-oscillatory behavior (the bottom scale). The equivalent transition step for the negative electron and negative up-quark occurs at a significantly lower *r*-value. Like the negative pi-meson, the decay rate is about the same for both the real and imaginary parts of both of those other particles – reaching as low as 10^{-15} GeV at the same distance as for their corresponding positive mass potentials. As the figure indicates, while both the electron and the up-quark are still oscillating (as *r* decreases), the amplitude of their potentials has reached the same magnitude as the negative pi-meson in its non-oscillatory increase. This difference in this behavior requires further investigation.

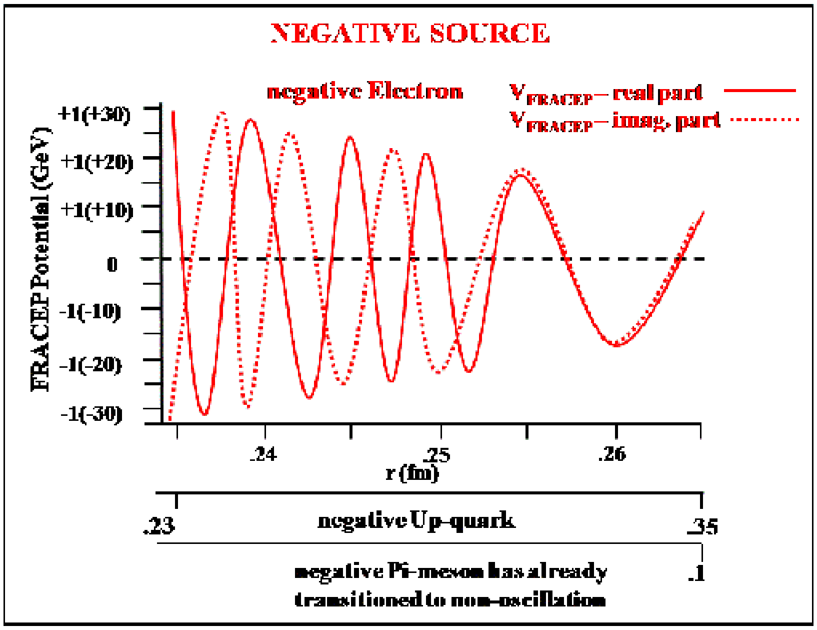

Figure 3: This shows the typical behavior of V_{FRACEP} for negative sources at the quantum through the fundamental scales. The plot shows the potential (real and imaginary parts) for the negative electron (the top scale). The bottom two scales show where the real part and the imaginary part of the negative up-quark and negative pi-meson have approximately the same values as the electron. In each case, m_s is negative and m_t equals the negative of m_s (positive).

CONCLUSIONS: This effort produced a unified potential (V_{FRACEP}) that characterizes the behavior for the fundamental particle through the composite versions of the Standard Model elementary particles. As developed, it was not intended to provide a model of specific phenomena – only an operational characterization of the overall field behavior for both positive and negative mass sources. In addition to the real and imaginary parts for the negative source (an effect not found in usual potentials like V_{Newton} or OPED), V_{FRACEP} also exhibits is a non-symmetric behavior between the positive source and the negative source. (Recall V_{Newton} simply replaces positive mass for negative mass, but the potential behaves the same except for a change in sign. This non-symmetric behavior in V_{FRACEP} requires further investigation; and, the full implication of it on any possible relation to dark matter has yet to be determined.

While the data used to develop the unified potential for the macro scale through quantum scale down to the pi-meson is readily available (for the positive mass sources), data for masses down to the FRACEP fundamental scale is not. With little guidance for defining a realistic correction for that smallest scale, the modifications to the basic potential (modifications to E_0 (b1), and the addition of E_1 (c1)) have been largely based on guesses and intuitive feelings for the correction forms.

One possible point of guidance comes from the work of Hoyle, et al. in their development of a canonical form of gravity equations that reduce to General Relativity under certain conditions [20]. Their theory indicates that during a creation event (like the Big Bang), pairs of particles with both positive mass and negative mass are created. The creation energy they compute is ~$6\text{x}10^{+18}$ GeV. In computing the potential for the negative source $\mathcal{G}n$, the potential energy for the real part of V_{FRACEP} was approximately that value at $r \sim 2\text{x}10^{-19}$ fm; and, the potential for the imaginary part was approximately that value at $r \sim 4\text{x}10^{-19}$ fm. This distance is about 10 radii for the $\mathcal{G}p$ and $\mathcal{G}n$ (and about 10 Planck lengths). This might imply that the pair of particles is created at a separation distance somewhere in the neighborhood of 10 radii apart, and, there is an explosive repulsive force between them at that time. One might speculate that the explosive force (driven by the repulsive potential – maybe the initial Dark Energy) is what fed the initial expansion of the universe at the moment of the Big Bang. An estimate of the early inflation energy, computed by Narlikar [21], is $E = 3.4\text{x}10^{16}\ \text{GeV} \cdot r^{1/4}$. For V_{FRACEP}, this early energy level is reached for both real and imaginary parts at r ~$7.5\text{x}10^{-19}$ fm. Further investigation of this possibility is needed.

Finally, this work shows there is a possibility that the $1/r$ Newtonian potential might represent a first order approximation to a more complicated function that unifies all the scales – even down to the fundamental Planck-length scale. This V_{FRACEP} function has characteristic behavior that is consistent with bright matter (positive mass) observations. But, it also allows for the characterization of non-traditional (negative mass) matter that may illuminate the puzzle of dark matter and energy – not just at cosmic scales, but also at Fermi scales.

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