

Article

Fractal Composite Quarks and Leptons with Positive and Negative Mass Components

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Abstract: The object of this work was to study the feasibility of identifying a minimum set of fundamental particles that could be used to build up composite versions of the Standard Model quarks, leptons and bosons that exhibit the same properties and behavior as the observations. The result is a fractal-based heuristic model that has only two fundamental particles (with equal and opposite mass). The fractal-like configurations of the multi-tiered substructure for the quarks and leptons demonstrated a dual universe – one based on mostly positive mass (the universe we see) and the other based on mostly negative mass (potentially a contributor to the cosmic dark matter universe we see evidence of without direct observation).

Keywords: composite quarks and leptons, sub-quark structure, sub-lepton structure, negative mass, fractal-based particles

1. Introduction

The current Standard Model of Particle Physics (SM) has proven to be a remarkably powerful description of nature at the quantum scales. It is a semi-empirical model with a theory based on quantum mechanics, and an observationally determined database. Despite the successes, its description of nature's fundamental matter and force particles is known to be incomplete, and exploring alternatives may help solve some of the difficulties. One such alternative is the FRACEP (Fractal Rings And Composite

Elementary Particles) model presented here, which describes the SM fermions and bosons as composite with both positive and negative mass components.

The SM theory treats its fermions and bosons as fundamental with no internal structure. Considering the SM analogously with the fundamental-atoms picture prevalent as recently as the mid nineteenth century, it is possible to hypothesize that a composite structure for the particles might resolve some of the theory's incompleteness. The hierarchical mass spectrum with its three-tiered generation structure raises the possibility of smaller building blocks of matter within the fermions. The mass-providing mechanisms of the Higgs scalar for the weak bosons, and the Yukawa coupling for the fermions requires a less than satisfyingly large number of free parameters in the theory. Also, it is aesthetically unpleasing that there is such a large number (~61) of fundamental particles rather than a small set.

The idea of compositeness in the SM particles began taking shape with preon theory (an attempt to describe particle substructure within the framework of a quantum mechanical formalism) since the 1970's. D'Souza and Kalman [1] provide good summaries: of the concerns of the completeness of the SM; of the issues leading to the interest in compositeness in the SM particles; of the composite models (on the order of 50) that evolved during the first two decades of development in the field; and of the experimental considerations for composite model validation. They note that most models are variants of the early well-known ones discussed. Many only address a single generation, but, some attempt to predict the mass spectrum for the three generations of leptons. Some are more heuristic, lacking discussion of dynamics, while most address the symmetry properties that make them consistent with the SM.

Most of the early models have several things in common: they contain a basic set of two to four preons with an equal number of anti-preons in each of the first generation particles. These basic units generally have intrinsic spin and charge (though the charge is generally $e/3$ or $e/6$), and mass generally is not considered intrinsic. The SM particles are built up from combinations of the basic units (with additional units as the generations increase). The models often considered preons as fundamental with no internal structure (for example, Harari [2] and Shupe [3]), but, others have addressed the possibility of compositeness in the preons (as in, 'tHooft [4], Terazawa and colleagues [5 - 7], and Nelson [8]). In [9], Nelson discusses the criteria for building plausible composite theories, and summarizes the progress to date in the field. She indicated that there has been a lot of activity and progress over the last 40 years, but no unique definitive theory yet.

Among the more well-known early models, Harari [2] observed that, because of their similarities, quarks and leptons should be constructed from the same basic units (preons in current nomenclature). His schematic model consisted of two basic spin $1/2$ preons (one with charge $Q = -1/3$, and one with $Q = 0$). His model also had two spin $-1/2$ anti-preons (one with $Q = +1/3$, and one with $Q = 0$). Each of his

simplest composite fermions had a unique combination of three basic preons, while the higher generations were hypothesized as excited states of the first-generation configurations. There was no explicit discussion of mass, and, he noted he presents no convincing dynamics.

Shupe [3] presented a similar picture in his heuristic model, developed independently. His model extends the picture to include three color states for quarks, and force-mediating bosons. He explicitly notes that his constituent preons and anti-preons have no intrinsic mass – assuming that the first generation masses represent self-energy of the photon and gluon fields coupling to the leptons and quarks. He noted that solitons have some of the desired properties for representing fermionic preon-anti-preon pairs.

A different approach to preons is considered here. Like Harari and Shupe, this model, FRACEP, is currently a heuristic model. It provides a description of a set of composite fermions and bosons that are consistent with the observed characteristics of the SM particles, but it uses a multi-tiered substructure in the lightest leptons consisting of preon level Intermediate Building Blocks (IBBs), pre-preon IBB component groupings and the fundamental particles that are their bases.

In the following sections we describe the philosophy of the FRACEP development (Section 2), followed by a discussion of the model's assumptions (Section 3). We then describe the structure of the IBBs and pre-IBBs (Section 4). Finally, we present the FRACEP composite structure for the fermions and bosons, comparing the composite and SM versions (Section 5). The heuristic nature of FRACEP includes a discussion of the zero-mass nature of photons and gluons. It does not include a formalized dynamic description of particle interaction, but it does include a conceptual description of quark confinement and color exchange. Much work is left to be done to more fully develop this model.

2. FRACEP Model Philosophy

The motivation for the FRACEP development was to determine the feasibility of constructing a composite structure for the fermions and bosons and their anti-counterparts. It had two major goals. One was to establish the basis of the substructure (for all particles and anti-particles) as a minimum set of fundamental particles. The second was that the observed properties be consistent with the SM observations in mass, spin, electromagnetic charge, and color charge.

To accomplish this, a multi-tiered composite structure was used. The larger-scale substructures, the IBBs, are roughly analogous to the more common preon concept, and are used to construct all of the SM fermions and bosons and their anti-counterparts in a more-or-less regular. The second-tier smaller-scale substructure assumes the IBBs are constructed from fractal-like groupings (the pre-IBBs), themselves constructed of the fundamental particles (the smallest tier).

Philosophically, FRACEP assumes that the larger-scale world mimics the fundamental structure of nature, and the observed larger-scale structure implies a similar form in the fundamental structure as well. Naturalness of this kind is an aesthetically pleasing way of seeing deeper into the structure of matter. Guidice [10] notes that such “aesthetic beauty is a powerful guiding principle for physicists as they try to construct new theories”.

Although it is broadly associated with gauge theory [4], from a more heuristic point-of-view, naturalness is easily visible in fractal structures. The self-similarity and iterative nature of fractals for different scales in nature was described and studied by Mandelbrot [11], considering the cosmic scales of galaxy distributions to the smaller scales of snowflakes and turbulence. Salam, et al. [12] developed a quark model assuming composite quarks consisting of preons which were also composite (pre-preons). Chang [13] extended this idea to develop a fractal model of particles. FRACEP addresses this idea in a more heuristic way, proposing possible structural components for the first generation SM particles [14]. This paper extends the model to all three generations of particles.

3. Requirements and Assumptions for the IBBs and Pre-IBBs

In FRACEP, we are seeking building blocks in the composite particles that are based on the smallest possible set of fundamental particles. FRACEP used the SM observations for its data set to define its composite equivalents. To guarantee the same decay path as the SM particles, it uses the observed decay paths as its definition for the necessary internal groupings. In this way, FRACEP’s construction satisfies the SM particles’ observed mass, spin, electric charge, color charge, and decay path.

One necessary requirement on the model development results from the theory indicating that particles and their anti-counterparts have the same mass. Observations appear to support the theory. This leads to two inequalities that must be satisfied by the IBB masses:

$$\text{mass}(v_{e-}) \cong \text{mass}(v_{e+}) < 15 \times 10^{-6} \text{ MeV}/c^2 \quad (1)$$

and

$$\text{mass}(e^-) - \text{mass}(e^+) < 4.0 \times 10^{-8} \text{ MeV}/c^2 \quad (2)$$

where $15 \times 10^{-6} \text{ MeV}/c^2$ and $4.0 \times 10^{-8} \text{ MeV}/c^2$ are the upper limit of the v_{e-} mass, and the measurement uncertainty in the e^- mass, respectively. Equations (1) and (2) are constraints in FRACEP on the masses of the spin and charge related components in the IBBs. Combining (1) and (2) with the definitions of the charge carriers (section 4.3), the spin carriers (section 4.4), and the electron and anti-electron (section

5.11, Table 4) defines the IBB masses and their component masses. These component masses establish the first generation particles.

A second requirement placed on the model is the need to describe the interaction of the fundamental particles, G_p and G_n , which has implications for the configuration of the momentum carriers (section 4.2) and other components as they are built up, and the half-life of the composite particles. This requirement was addressed by developing a unified interaction potential [15] that extends the fractal concept, employing the idea that potential fields of large masses follow the same fundamental rules as the small masses. The FRACEP potential, V_{FRACEP} , has the characteristic $1/r$ behavior at the largest macro scales of Newton

$$V_N = -Gm_t m_s / r, \tag{3}$$

and, the decaying oscillatory behavior at the quantum scales of the modified Yukawa potential for nuclear scattering

$$V_{Y-mod} \sim -g^2 e^{-br/r} \times F(r, m). \tag{4}$$

V_{FRACEP} takes the form

$$V_{FRACEP} = [A_0(m) + B_0(\sqrt{m})] \times \sin[S(r, m) + T(r, \sqrt{m})] \times \exp^{-E(r, m)}. \tag{5}$$

Extending down to the scales of G_p and G_n , well below the valid range of (4), it retains the oscillatory behavior characteristic of the same kind of particle containment observed at the nuclear scales.

A related imposed requirement is the need to be able to uniquely structurally distinguish between particles and their anti-counterparts. The square-root of mass in (5) satisfies this need. As expected, V_{FRACEP} shows a repulsive force between the positive mass G_p and the negative mass G_n . This naturally leads to instability in the mixed-mass composite leptons and quarks.

3.1. Assumption 1: Existence of Negative Mass

In its most fundamental state, matter in the universe is polarized into only two types with energy potentials that are interpreted as mass. That assumption leads to a minimum set of only two fundamental particles that have equal and opposite intrinsic mass (positive mass G_p , and negative mass G_n). The idea of negative mass is not a new one theoretically. For example, it is developed from the perspective of dual or multiple universes [16, 17], and is considered from the perspective of V_{FRACEP} 's unified gravitational potential for positive and negative mass [15]. FRACEP differs from the more traditional preon models which de-facto assume only positive mass whether intrinsic or gained through field

interactions. The introduction of the negative mass components is used to account for observed particle instability.

3.2. Assumption 2: Electric Charge

Electric charge is a component (a charge carrier which is also assumed to be composite), and, the observed electric charge is a dynamical effect of the carrier. There are two possible fundamental charge values: $+1/3e$ and $-1/3e$; and, the total charge on any particle is the sum of several charge carriers. (Because of their fundamental nature, the G_p and G_n have zero charge.)

This fractional charge summation leads to a total charge that agrees with the SM observed charge for all particles. In contrast, the SM assumes electric charge is an inseparable, inherent property of any particle, with a fundamental value of $-1e$ (the charge on its electron family). This fundamental charge is then rotated through a phase angle to produce the fractional charges on the two quark families.

3.3. Assumption 3: Spin

Similarly, spin is a component (a spin carrier which is also assumed to be composite), and, the observed spin is a dynamical effect of the carrier. The spin carrier has two values: $+1/2$ and $-1/2$; and, the total spin on any particle is the sum of several spin carriers which can align parallel or anti-parallel. (Because of their fundamental nature, the G_p and G_n have zero spin.) This spin summation leads to a total spin that agrees with the SM observed spin for all particles. Like electric charge, the SM assumes spin is an inseparable, inherent property of its fermions ($\pm 1/2$) or its bosons (0 or ± 1).

3.4. Assumption 4: Color Charge

Color charge is a component in the quark construction. There are 3 color charges and 3 anti-color charges leading to 8 gluons with \sim zero mass (like the SM). A color charge is a component in the up-quark family members, but (unlike the SM) the down-quark family members get their color charge from the related up-quark which is part of their construction. This construction leads naturally to an observed color exchange mechanism that is part of the quark confinement in nucleons.

3.5. Assumption 5: Particle Size

The fundamental particles, G_p and G_n , have a finite size (the Planck length). Because the composite structure of the leptons and quarks contains an increasing number of “finite sized” components (built up from the G_p 's and G_n 's), a size distribution with increasing mass naturally arises

($\sim 10^{-25}$ m for ν_{e-} to $\sim 4 \times 10^{-19}$ m for the bottom quark) – consistent with the SM estimate of the largest possible size $< 10^{-18}$ m (based on the inverse of the observed scattering cross-section). The SM typically represents its particles as point sources.

3.6. Assumption 6: Matter-Anti-Matter Mass Difference

With the exception of FRACEP's composite electron and electron neutrino, all of its composite particles contain components with both positive and negative mass elements. This leads to a small difference between the particle and its anti-particle in FRACEP's "Bright Universe" (the observed SM particles). The SM theory predicts there should be no difference between the two. Precise measurements of the electron and the anti-electron masses might show the difference FRACEP predicts.

The ALPHA collaboration at CERN [18] is considering the negative-mass question from the point-of-view of possible gravitational repulsion in anti-hydrogen. Specifically, assuming the hydrogen atom is purely positive mass, is the anti-hydrogen atom purely positive (with opposite characteristics), or is it partly, or totally, negative mass causing it to violate the expected gravitational behavior? Their initial efforts [19] created sufficiently stable anti-hydrogen, and further experiments are planned to more completely answer the question.

3.7. Assumption 7: Zero-Mass Bosons (Photon and Gluons)

The zero-mass bosons are "finite sized" composite particles that attribute their zero mass to the equal amounts of positive mass and negative mass components. They are assumed to represent a bridge between the FRACEP "Bright Universe" (particles based primarily on positive mass) and the FRACEP "Dark Universe" (a parallel set of particles based primarily on negative mass). The SM treats these bosons as fundamental, with no internal components, and, it has no intuitive explanation for the nature of a physical particle with zero mass. (The SM is also struggling with the nature of dark matter in general – the dark side of cosmic particles which FRACEP's "Dark Universe" may help to explain.

4. The IBB and Pre-IBB and Fundamental Particles

A fractal-like configuration is used to build up three types of Intermediate Building Blocks, the IBB's (momentum carriers, charge carriers, and spin carriers). These IBBs are defined as intermediate because their mass and structure lie between the smallest (fundamental) particles (G_p and G_n) from which they are constructed, and the larger composite versions of the SM fermions and bosons which they are used to construct. FRACEP assumes G_p has a classical radius, $r(G_p)$, equal to the Planck length.

4.1. The Fundamental Particles

The G_p and G_n are assumed to have charge = 0 and spin = 0, with masses that are equal and opposite. To determine the mass of G_p , we consider the creation event presented in Hoyle, et al. [20]. According to their theory, particle creation would occur with an “opening-up” of space-time which is precipitated at the energy associated with a Planck particle, $\sim 6 \times 10^{18}$ GeV. Once created, the massive particle, m_P , then decays to smaller energy particles. In typical quantum-gravity theory, the gravity model is not scale invariant. The mass of m_P is determined by setting the deBroglie wavelength equal to the Schwarzschild radius

$$\frac{2\pi \cdot \hbar}{m \cdot c} = \frac{2G \cdot m}{c^2} \tag{6}$$

This gives a mass of the Super-GUT unification as

$$m_P = \sqrt{\pi \cdot (\hbar \cdot c / G)^{1/2}} \cong 1.22 \times 10^{22} \text{ MeV}/c^2. \tag{7}$$

Hoyle used a scale-invariant cosmology-gravity model, and determined this mass, m_{P^*} , as when the Compton wavelength equals the Schwarzschild radius giving

$$m_{P^*} = (3/4\pi)^{1/2} \cdot (\hbar \cdot c / G)^{1/2} \cong 5.97 \times 10^{21} \text{ MeV}/c^2. \tag{8}$$

FRACEP hypothesizes that symmetry in nature manifests itself such that the largest mass particle (m_{P^*}) has a reciprocal relation to the smallest mass particle (FRACEP’s G_p). The best fit of $G_p \cong 1/m_{P^*}$, allowing the first generation particle masses to satisfy the observations within the measurement uncertainty, was determined for a mass of G_p that is $m(G_p) = +1.724934 \times 10^{-22} \text{ MeV}/c^2$ and $m(G_n) = -m(G_p)$. The classical radius of G_p was assumed to be Hoyle’s scale-invariant Planck length, $r(G_p) = 3.307519 \times 10^{-35} \text{ m}$.

The introduction of the negative mass particle, G_n , was done by necessity. Without it, the composite model shows no distinctive structural difference that distinguishes particles (e.g., electron, e^-) from their anti-particles (e.g., anti-electron, positron, e^+) – that is, there is no difference between a positive spin (or negative charge) and a negative spin (or positive charge) component. The negative mass particle also provides an intuitive explanation for the instability in the leptons.

4.2. The IBB: Momentum Carriers

The momentum carriers (the first IBB type) are composed only of the fundamental particles, and have zero spin and zero charge. The G_p and G_n clump in fractal-like stable ring structures to form

momentum-particle carriers, MGXp(n), and momentum-ring carriers, MRXp(n). The “X” indicates the fractal level of the carrier; the “G” indicates it is a “general” momentum-particle carrier used to build up all of the composite particles; and the “p” or “n” indicate the mass is either all positive or all negative respectively.

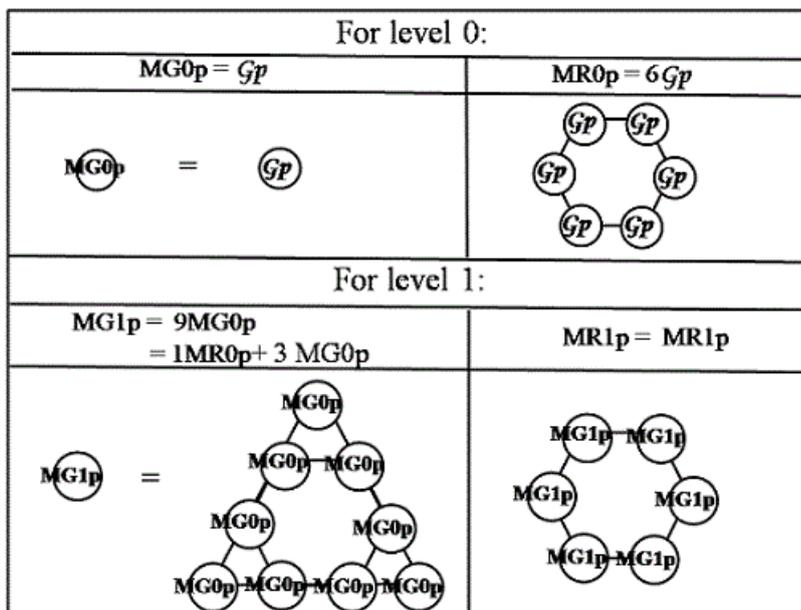


Figure 1. This shows the fractal structure of the positive mass general Momentum-Particle Carriers (MGXp) and the Momentum-Ring Carriers (MRXp) for the smallest structures (level 0) and the next higher fractal level 1. Level 2 would replace MG0p with MG1p in all elements of the level 1 structure. The negative mass particles and rings replace Gp with Gn and the names would become MG0n, MR0n, MG1n, MR1n,

The MGXp(n) particle is a 6-element ring plus three additional elements (Figure 1). The number of particles at each fractal level is 9^X , with a mass of

$$m(MGXp) = m(Gp) \cdot 9^X \quad (\text{for a positive mass particle}). \tag{9}$$

The m(MGXn) replaces Gp with Gn in (9) for a negative mass particle. The mass of the positive ring MRXp is $m(MRXp) = 6 \cdot m(MGXp)$. Hereafter, for convenience of notation, the particle name will also designate the particle mass.

The 6-element ring structure is reminiscent of the ubiquitous benzene ring structure in chemistry. The stability of the ring structure is considered to be the result of the structure symmetry. Like the benzene ring, the FRACEP rings are perfectly regular hexagons with each bond angle being 120° . All of the separation distances between pairs of elements along the ring are exactly the same length. (All elements on the ring have the same fractal level).

The closed ring structure is conjugated with both single and double bonds giving three binding sites per element. Each element shares a double bond with only one other element. To form a particle, the double bonds in the ring are broken, and the two adjacent particles previously sharing the double bond, now bond to an additional particle, producing a 9-element structure. To maintain equal separation between all pairs of particles, the three additional elements must be out of the plane of the ring. Two of the particles are above the ring plane and one is below. Geometric computations show the particle occupies a roughly spherical volume at each level.

4.3. The Charge Carriers

The charge carriers (the second IBB type) are components in all of the composite particles with the exception of the electron-neutrino, the anti-electron-neutrino and their dark (negative mass) counterparts. There are four charge carriers (QBp, QBn, QDp, and QDn), which have zero spin and a charge of $+1/3e$ or $-1/3e$. The carriers have total net positive or net negative mass (Table 1).

Table 1. This shows the masses of the charge and spin carrier IBB’s for the “Bright Universe” and the “Dark Universe”. All masses are in MeV/c^2 .

	BRIGHT UNIVERSE	DARK UNIVERSE
Charge Carrier IBB (Spin = 0)	QBp($q = -1/3e$) $m = 4.651945 \times 10^{-4}$	QDn($q = +1/3e$) $m = -4.651945 \times 10^{-4}$
Anti-Charge Carrier IBB (Spin = 0)	QBn($q = +1/3e$) $m = 4.651941 \times 10^{-4}$	QDp($q = -1/3e$) $m = -4.651941 \times 10^{-4}$
Spin Carrier IBB (Charge = 0)	SBp($s = +1/2$) $m = 1.278588 \times 10^{-6}$	SDn($s = -1/2$) $m = -1.278588 \times 10^{-6}$
Anti-Spin Carrier IBB (Charge = 0)	SBn($s = -1/2$) $m = 1.278482 \times 10^{-6}$	SDp($s = +1/2$) $m = -1.278482 \times 10^{-6}$

Each of the charge carriers is composed of two charge-specific pre-IBB’s: 1) a charge-momentum part, MQp(n), that carries the bulk of the mass but no charge; and, 2) a charge-effect part, QEp(n), that carries the dynamic structure producing the observed charge but only a small amount of mass (Table 2). The four carriers are defined as follows (with all masses in MeV/c^2).

One, QBp = MQp + QEp. It has all positive mass components, spin = 0, and charge = $-1/3e$. A stable configuration containing 3QBp’s, leads to what FRACEP calls its “Bright Universe” electron (the electron of the SM).

Two, QBn = MQp + QEn. The mass of QBn equals the mass of QBp – δq , where $\delta q = 3.793170 \times 10^{-10}$. It has a positive mass charge-momentum pre-IBB and a negative mass charge-effect

pre-IBB, spin = 0, and charge = +1/3e. A configuration containing 3QBn's is unstable and leads to FRACEP's "Bright Universe" anti-electron (the anti-electron of the SM).

Table 2. This shows the pre-IBB's for the charge and spin carriers. The charge is in units of electron charge, the spin is in units of h-bar, and the mass is in units of MeV/c². As defined in Table 1, the four charge carriers have one charge-momentum and one charge-effect pre-IBB, and the four spin carriers have one spin-momentum and one spin-effect pre-IBB.

CHARGE AND SPIN CARRIERS (IBB's) COMPONENTS			
	Mass	Spin	Charge
CHARGE CARRIER PRE-IBB's			
Charge-Effect pre-IBB's			
QEp (Positive mass)	1.896585x10 ⁻¹⁰	0	-1/3
QEn (Negative mass)	-1.896585x10 ⁻¹⁰	0	+1/3
Charge-Momentum pre-IBB's			
MQp (Positive mass)	4.651941x10 ⁻⁰⁴	0	0
MQn (Negative mass)	-4.651941x10 ⁻⁰⁴	0	0
SPIN CARRIER PRE-IBB's			
Spin-Effect pre-IBB's			
SEp (Positive mass)	5.264081x10 ⁻¹¹	+1/2	0
SEn (Negative mass)	-5.264081x10 ⁻¹¹	-1/2	0
Spin-Momentum pre-IBB's			
MSp (Positive mass)	1.278535x10 ⁻⁰⁶	0	0
MSn (Negative mass)	-1.278535x10 ⁻⁰⁶	0	0

Three, QDn = MQn + QEn. The mass of QDn equals the negative of the mass of QBp. It has all negative mass components, spin = 0, and charge = +1/3e. A stable configuration containing 3Qn's, leads to what FRACEP calls its "Dark Universe" dark-electron – a negative mass particle not considered in the SM, but may be a contributor to the cosmic dark matter being sought.

Four, QDp = MQn + QEp. The mass of QDp equals the mass of QDn + δq. It has a negative mass charge-momentum pre-IBB and a positive mass charge-effect pre-IBB, and charge = -1/3e. A configuration containing 3QDp's is unstable and leads to FRACEP's "Dark Universe" dark-anti-electron – again not found in the SM.

The MQp and QEp are all positive mass; and, the MQn and QEn are all negative mass – so the four combinations give the charge carriers: 1) all positive mass, 2) all negative mass, or 3) a mixture of positive and negative mass. It is the carriers with a mixture that allow for the distinguishing features

between matter and anti-matter, and it causes the mass particles to have a slightly larger mass value than their respective anti-particles. Also, the mixed-mass type drives the composite particles to decay.

4.4. The Spin Carriers

The construction of the spin carriers (the third IBB type) is parallel to the charge carrier construction. Spin carriers are components in all of the composite particles without exception.

There are four spin carriers (SBp, SBn, SQDp, and SDn), which have zero charge and a spin of $+1/2$ or $-1/2$. The carriers have total net positive or net negative mass (Table 1). Each of the spin carriers is composed of two spin-specific pre-IBB's: 1) a spin-momentum part, MSp(n), that carries the bulk of the mass but no spin; and, 2) a spin-effect part, SEp(n), that carries the dynamic structure producing the observed spin but only a small amount of mass (Table 2).

Like their charge-specific pre-IBB equivalents, the MSp and SEp are all positive mass, and the MSn and SEN are all negative mass, giving the spin carriers: 1) all positive mass, 2) all negative mass, or 3) a mixture of positive and negative mass – with the same consequences on the particle behavior as the charge carriers. The four carriers are defined as follows (with all masses in MeV/c^2).

One, SBp = MSp + SEp. It has charge = 0, spin = $+1/2$, and all positive mass components – a stable configuration in the “Bright Universe” (“BU”) particles and anti-particles.

Two, SBn = MSp + SEN. The SBn mass = (SBp – δ_s) mass, where $\delta_s = 1.0528162 \times 10^{-10}$. It has charge = 0, spin = $-1/2$, and, a positive mass spin-momentum pre-IBB and a negative mass spin-effect pre-IBB – an unstable configuration in the “BU”.

Three, SDn = MSn + SEN. The SDn mass = –SBp mass. It has charge = 0, spin = $-1/2$, and all negative mass components – a stable configuration in “Dark Universe” (“DU”) particles and anti-particles.

Four, SDp = MSn + SEp. The SDp mass = (SDn + δ_s) mass. It has a negative spin-momentum pre-IBB and a positive (mass) spin-effect pre-IBB with a spin of $+1/2$ – an unstable configuration in the “DU”.

5. The FRACEP Composite Particles

Having provided the assumptions and the definitions of the G_p and G_n , the IBB and pre-IBB building blocks, we now proceed to describe the composite structures that we propose to reproduce the observations of the SM elementary particles and anti-particles.

Table 3. This compares the computed FRACEP composite fermion masses with the SM measured fermion masses. Column 1 indicates the particle with its charge in parentheses. Column 2 is the computed mass for the FRACEP particles. Column 3 is the mass error ($\epsilon = \text{FRACEP} - \text{SM}$) with the SM mass uncertainty, Upper Bound, or Range (δ) in parentheses. Column 4 is the FRACEP Δ -mass (particle – its anti-particle). The SM Δ -mass = 0. Masses are in units MeV/c^2 .

FIRST GENERATION PARTICLES (spin = 1/2)			
	FRACEP Mass	Mass Error ϵ (SM δ)	FRACEP Δ-Mass
ν_{e^-} (0)	1.278588×10^{-6}	$< 15 \times 10^{-6}$ (SM UB)	$1.05281620 \times 10^{-10}$
e^- (-1)	0.510999	-2.01×10^{-8} ($\pm 4.0 \times 10^{-8}$)	$1.45379597 \times 10^{-9}$
u^+ (+2/3)	3.568130	in SM R (1.5–4.5)	$-8.63915162 \times 10^{-10}$
d^- (-1/3)	5.098330	in SM R (5.0–8.0)	$4.84599916 \times 10^{-10}$
SECOND GENERATION PARTICLES (spin = 1/2)			
ν_{μ^-} (0)	0.169868	< SM UB (<0.17)	1.052816×10^{-10}
μ^- (-1)	105.658357	-2.01×10^{-8} ($\pm 4.9 \times 10^{-8}$)	1.453785×10^{-9}
c^+ (+2/3)	1225.390018	in SM (1100–1400)	$-8.637926 \times 10^{-10}$
s^- (-1/3)	124.686299	in SM R (88–155)	4.845901×10^{-10}
THIRD GENERATION PARTICLES (spin = 1/2)			
ν_{τ^-} (0)	3.738119	< SM UB (<18.2)	$1.0528116 \times 10^{-10}$
τ^- (-1)	1777.053018	1.18×10^{-4} ($\pm 1.6 \times 10^{-4}$)	1.453827×10^{-9}
t^+ (+2/3)	182976.620924	in SM R (172800 – 183200)	$-8.440111 \times 10^{-10}$
b^- (-1/3)	4399.212107	in SM R (4100 – 4400)	4.838512×10^{-10}

5.1. The Fermions

The FRACEP composite particles are identical in their outward characteristics (mass, spin, electromagnetic charge) with the SM elementary fermions and anti-fermions (Table 3). The only exception is the anti-particles which have a slightly smaller mass than their corresponding particle because of the negative mass components in the charge carriers and spin carriers that are needed to change the sign of the charge and spin. Color charge is addressed below with the bosons (section 5.2).

5.1.1. The “Primary Composites”

The IBB’s are the basic units that are used to build all of particles in both the “Bright Universe” (“BU”) and the “Dark Universe” (“DU”). FRACEP has 12 “primary composites” (Table 4) which are the simplest structures and the smallest mass particles. They include ν_{e^-} and e^- (two first generation

particles) and $\nu_{\mu-}$ (the second generation neutrino), their anti-particles, and their “DU” counterparts. The “primary composites” are components in the next larger particle group (the “secondary composites”).

The “BU” composites are primarily based on positive mass; but, the parallel set of particles in the “DU” is based primarily on negative mass. The “DU” particles are not the anti-particles of the “BU” particles. Only the ν_{e-} , $e-$, $D-\nu_{e+}$, and $D-e+$ are stable because they are not mixed-mass, that is, they contain only all positive or all negative mass. All of the other particles (and antiparticles) in the “BU” and “DU” are unstable. It is hypothesized that in the “BU” the negative mass components are surrounded by a halo of the positive mass momentum groupings that affect the time with which decay occurs. Three-dimensional Brownian motion simulations can provide estimates of the half-life of the particles [21].

Table 4. This shows the composition of the “primary composite” fermions and anti-fermions for both the “BU” and “DU”. The elements in “green” have all positive mass and are stable. The elements in “red” have all negative mass and are stable. The elements in “black” are a mixture of positive and negative mass and are unstable. The $e-$ and $D-e+$ have negative charge, and the $e+$ and $D-e-$ have positive charge. The other particles in this group have charge = 0.

BRIGHT UNIVERSE		DARK UNIVERSE	
“Primary Composite” Particles: (“BU” has spin = +1/2, “DU” has spin -1/2)			
ν_{e-}	SBp	$D-\nu_{e+}$	SDn
$\nu_{\mu-}$	$(\nu_{e-} + QBp + MG22p) + QDn$	$D-\nu_{\mu+}$	$(D-\nu_{e+} + QDn + MG22n) + QBp$
$e-$	$3(\nu_{e-} + QBp + MG22p)$	$D-e+$	$3(D-\nu_{e+} + QDn + MG22n)$
“Primary Composite” Antiparticles:(“BU” has spin = -1/2, “DU” has spin +1/2)			
ν_{e+}	SBn	$D-\nu_{e-}$	SDp
$\nu_{\mu+}$	$(\nu_{e+} + QBn + MG22p) + QDp$	$D-\nu_{\mu-}$	$(D-\nu_{e-} + QDp + MG22n) + QBn$
$e+$	$3(\nu_{e+} + QBn + MG22p)$	$D-e-$	$3(D-\nu_{e-} + QDp + MG22n)$

5.1.2. The “Secondary Composites”

The next larger (and more complex) group of particles is the “secondary composites” (Table 5). They include $u+$ and $d-$ (two first generation particles) and $\nu_{\tau-}$ (the third generation neutrino), and their anti-particles, and “DU” counterparts. All particles in this group are unstable because they contain mixed-mass elements.

Table 5. This shows the composition of the “secondary composite” fermions for both the “BU” and “DU”. Positive mass is in “green”; negative mass is in “red”; and mixed-mass is in “black”. All of the particles in this group are mixed-mass and are unstable. The u+ and D-u- have positive charge, and d- and D-d+ have negative charge. Their corresponding anti-particles have the opposite charge. The neutrinos have charge = 0. The base unit in “BU” is {2MR22p}, and {2MR22n} in “DU”.

BRIGHT UNIVERSE		DARK UNIVERSE	
“Secondary Composite” Particles: (“BU” has spin = +1/2, “DU” has spin = -1/2)			
u+	2(v _{e+} + QBn + MG22p) + {2MR22p} + (MR22p + v _{μ-}) + CQ(i)	D-u-	2(D-v _{e-} + QDp+ MG22n) + {2MR22n}+ (MR22n + D-v _{μ+}) + CQ(i)
v _{τ-}	(v _{μ-} + MG22p) + {2MR22p} + R _{2v_μ}	D-v _{τ+}	(D-v _{μ-} + MG22n) + {2MR22n} + D-R _{2v_μ}
d-	u+ + R _{d-}	D-d+	D-u- + D-R _{d+}
Radicals Definitions:			
R _{d-} = v _{e+} + MR22p + e-; R _{2v_μ} = v _{μ-} + MR22p + v _{μ+}		D-R _{d+} = D-v _{e-} + MR22n + D-e+; D-R _{2v_μ} = D-v _{μ+} + MR22p + D-v _{μ-}	

The “primary composites” are components in their structure, but they also have a small base unit ({2MR22p(n)} in “BU” or (“DU”)) which is not present in the “primary composites”. Four radicals are also included among their components: 1) R_{d-} = **MR22p** + v_{e+} + e-; 2) R_{2v_μ} = v_{μ-} + **MR22p** + v_{μ+}; and their two “DU” counterparts. These radicals are used by the up-quark family members to build the corresponding down-quark family. Each quark contains one of three possible color charges “CQ(i)” – giving a total of 28 distinct particles in the group including anti-particles and “DU” particles. The color charge structure is discussed in Section 5.2 with the composite bosons. All anti-particles are formed in the same way as the “primary composites”.

5.1.3. The “Heavy Composites”

The last and largest (and most complex) group of particles is the “heavy composites” (Table 6). They include the μ-, c+ and s- (second generation particles) and τ-, t+ and b- (third generation particles). The “primary composites” and the “secondary composites” are components in their structure.

The “heavy composites” have a larger base unit than the “secondary composites”, one or two (2x{3MR22p(n)} units). The dynamic spin and charge components for the particle are defined in terms of the simpler composites; but, unlike the lighter composites, the “heavy composites” also have a momentum carrying grouping – large clumps of MRXp(n)’s carrying the bulk of the particle’s mass. For

the “BU” particles (composite versions of the SM particles), these clumps are all positive mass. For the “DU”, these clumps are all negative mass.

Table 6. This shows the composition of the “heavy composites” for the “BU”. Positive mass is in “green”; negative mass is in “red”; and mixed-mass is in “black”. All of the particles in this group are mixed-mass and are unstable. The μ^- and τ^- have charge = $-1e$; c^+ and t^+ have $+2/3e$ charge; and s^- and b^- have $-1/3e$ charge. Column 2 shows the core grouping (base plus dynamic components), and column 3 shows the bulk momentum carrying grouping which have spin = 0 and charge = 0.

BRIGHT UNIVERSE		
“Heavy Composite” Particles: (“BU” has spin = +1/2)		
	CORE	BULK MOMENTUM
μ^-	$(MR22p + v_{\mu^-}) + (v_{e^+} + MR22p + QBn) + \{2 \times 3MR22p\} + MR22p + R_{d^-}$	93MR22p
c^+	$(R_{u^+} + MR22p + R_{u^-}) + \{2 \times 3MR22p\} + R_{u^+} + R_{v_{\tau^-}}$	13MR24p + 53MR22p
s^-	$c^+(\text{core}) + R_{d^-}$	---
τ^-	$(R_{u^+} + MR22p + R_{u^-}) + \{2 \times 3MR22p\} + R_{\tau^-} + R_{v_{\tau^-}} + \{2 \times 3MR22p\} + 2MR22p + R_{d^-}$	93MR22p + (17MR24p + 76MR22p + 536MR19p + 120MR16p)
t^+	$(R_{c^+} + MR22p + R_{c^-}) + \{2 \times 3MR22p\} + R_{c^+} + R_{v_{\tau^-}}$	25MR26p + 57MR24p +12MR22p
b^-	$t^+(\text{core}) + R_{d^-}$	---
Radicals Definitions:		
$R_{d^-} = v_{e^+} + MR22p + e^-$	$R_{2v_{\mu}} = v_{\mu^-} + MR22p + v_{\mu^+}$	
$R_{u^+} = 2MR22p + v_{\mu^+} + u^+$	$R_{u^-} = 2MR22p + v_{\mu^-} + u^-$	
$R_{c^+} = 2MR24p + v_{\tau^+} + c^+$	$R_{c^-} = 2MR22p + v_{\tau^-} + c^-$	
$R_{\tau^-} = v_{\tau^+} + MR22p + \tau^-$	$R_{v_{\tau^-}} = MR22p + MR24p + v_{\tau^-}$	

The anti-particles and the “DU” counterparts of the “heavy composites” are formed in the same way as the primary composites. There are a total of 56 particles in all. The bulk momentum groupings make the total masses of c^+ and t^+ significantly greater than their corresponding down-quark pairs, s^- and b^- , a reversal of the pattern in the first generation.

With only the core masses, the pattern is consistent across the three generations (Table 7). This hints at the possibility that the observed c^+ and t^+ are excited states of very short-lived core particles, with the bulk momentum providing the excitation energy.

Table 7. This shows relative masses of the up-quark and down-quark families. Considering just the core mass, the FRACEP up-quarks are consistently less than the down-quarks in all generations.

	FRACEP MASS		SM OBSERVED MASS
	Core	Core + Bulk Momentum	
u+	3.568130	same	1.5–4.5
d–	5.098330	same	5.0–8.0
c+	123.156011	1225.390018	1100–1400
s–	124.686299	same	88–155
t+	4397.681908	182976.62092	172800 – 183200
b–	4399.212107	same	4100 – 4400

5.2. The Bosons

FRACEP considers the bosons and anti-bosons as a bridge between the “BU” and the “DU”. Only the zero-mass bosons are considered here. (The heavy bosons are not addressed at this time.)

5.2.1. The Photons

The zero mass of the photon results from its having components with equal amounts of positive and negative mass. FRACEP considers it a connection between the “BU” and the “DU” as it mediates between the fermions and the electromagnetic field. It represents an interaction between a “BU” electron (e^-) and a “DU” electron ($D-e^+$), with a proposed construction for the photon particle, γ , as:

$$\gamma = \{e^-, D-e^+\}.$$

Recall from Table 4, the composition of e^- is $3(v_{e^-} + QBp + MG22p)$, which is totally positive mass. The composition of $D-e^+$ is $3(D-v_{e^+} + QDn + MG22n)$, which is totally negative mass. Both particles are completely stable because they contain no mixed-mass components.

In the γ , the opposite masses of the two particles tend to repel each other, while the opposite charges tend to attract, leading to the formation of a very short-lived bond. The net mass, spin and charge are all equal zero during the interaction – making it undetectable as a traditionally defined particle.

FRACEP also has an anti-photon particle with zero net mass, spin and charge, but its mixed mass components are not stable.

$$\gamma^* = \{e^+, D-e^-\}.$$

Neither γ nor γ^* are long-term stable because their two components are of opposite mass.

The two structures, γ and γ^* could be considered virtual photons (popping in and out of existence) that store the interaction energy in the bond between the particles of the pairs. When their bonds break,

the energy is released to vibrate the fabric of space, the light we see and call the photon (particle/wave) entity. We propose the following scenario.

Space is teeming with “BU” e^- and “DU” $D-e^+$. During their free states, the e^- is detectable, but $D-e^+$ cannot be seen. When the two free particles cross paths, the e^- pops out of existence as the bond is formed, and the pair becomes undetectable because they have all properties equal to zero. When the brief bond is broken, the e^- pops back into existence and is again detectable.

When the γ pair-bond breaks, there are two vibrations released. One has positive frequency (the light of the bright photon we see), and the other has negative frequency (the dark-light of the dark photon which we would not see). These two photons would not travel at the same speed because of the energy difference. Note that the concept of negative frequency has no analog in the SM, and requires further investigation to explain its interpretation.

Similarly, when the γ^* (anti- γ) pair-bond is broken there are two vibrations released: one positive frequency (another bright photon) and one negative frequency (another dark photon). In this case, the positive frequency photon from the γ would contain higher energy than the positive frequency photon from the γ^* because of the small negative mass content in the anti-electrons.

So not only would the bright and dark photons from the γ have different speeds, but the bright and dark photons from the γ^* would also have different speeds. Further, the bright photons from the γ would have a different speed from the bright photon from the γ^* – although they would not be different by much.

The idea of different speed photons in a dual universe was developed in the context of the Janus cosmological model [17] – though it was not directed toward composite photons of mixed (positive and negative) mass.

5.2.2. The Gluons

FRACEP considers the gluons and anti-gluons to be composite – bridging the “BU” and the “DU” as they mediate between the quarks and the strong field through color-charge exchange. We propose that the color-charges represent the interaction between “BU” and “DU” neutrinos. Like the γ , the color-charge is not long-term stable. The proposed construction for the color-charge is: red = $\{v_{e^-}, D-v_{e^+}\}$ (with mass = 0); green = $\{v_{e^-}, D-v_{e^-}\}$ (with mass = 1×10^{-10}); and blue = $\{v_{e^-}, v_{e^+}\}$ (with mass = 2.5×10^{-6}).

The SM considers gluons as fundamental, having one color-charge and one anti-color-charge. The eight combinations are: rg^* , rb^* , gr^* , gb^* , br^* , bg^* , plus two mixed states ($\sqrt{1/2} (rr^* - gg^*)$ and $\sqrt{1/6} (rr^* + gg^* - 2bb^*)$). The tacit assumption here is that the color- and anti-color-charges are inherent

characteristics of the gluon, so, mixed states are mathematical expressions of the color state. To date, there are no data interpreted as indicating internal components in the SM gluon.

In a different approach, Guido [22] hypothesizes an internal geometrical structure of quantum oscillators within particles that give rise to particle properties such a color-charge. His system does not imply internal components – the properties are the manifestation of an additional internal degree of freedom provided by the linked oscillators.

FRACEP assumes the gluons and color-charge are linked components. Like the SM, it also has eight gluons, but, their composite structures are formed by combining a base component, plus one color-charge component, CQ(i), and one anti-color-charge component, CQ*(j). The 8 combinations are: (1) gr*; (2) gb*; (3) rg*; (4) bg*; (5) br*-1; (6) rb*-1; (7) br*-2; (8) rb*-2. The “-1” and “-2” indicate different base components connecting the gluon-anti-gluon pairs.

Since color-charge is a component attached to a base gluon, FRACEP cannot physically realize the mixed mathematical states of the SM. Instead, to achieve the eight gluon combinations, there are several varieties of the base gluons. The combinations of r, r*, b, and b* can have two different structures depending on the base which guarantees total masse ~zero ($<10^{-10} \text{ MeV}/c^2$) for the gluon structure (Table 8).

Table 8. This shows the five gluon base components and masses. Column 4 shows the color-charge and anti-color-charge attached to that base. Column 5 shows the net mass of the gluon (base + CQ + CQ*).

Gluon Bases	Base Components	Base Mass (MeV/c ²)	Attached CQ, CQ*	Total Gluon Mass (MeV/c ²)
G₁	(D-v _{e-} + R22p) + (R22m + D-v _{e+})	-2.5x10 ⁻⁶	g, r*	0
			g, b*	+1x10 ⁻¹⁰
G₂	(v _{e-} + R22p) + (R22m + v _{e+})	+2.5x10 ⁻⁶	r, g*	0
			b, g*	-1x10 ⁻¹⁰
G₃	(v _{e+} + R22p) + (R22m + D-v _{e+})	-1x10 ⁻¹⁰	b, r*(1)	0
G₄	(v _{e-} + R22p) + (R22m + D-v _{e-})	+1x10 ⁻¹⁰	r, b*(1)	0
G₅	(R22p) + (R22m)	0	b, r*(2)	+1x10 ⁻¹⁰
			r, b*(2)	-1x10 ⁻¹⁰

Based on this construction, we see that the gluon bases G1, G2 and G5 are their own anti-base; and G3 is the anti-base of G4. Note that the gluon {G1, gr*} is the anti-grouping of {G2*, g*r}, and {G1, gb*} is the anti-grouping of {G2, g*b}.

The $\{G3, br^*\}$ is the anti-grouping of $\{G4, b^*r\}$; and $\{G5, br^*\}$ is the anti-grouping of $\{G5, b^*r\}$. This means that the eight gluon grouping set is its own set of anti-groupings, and there is no independent anti-grouping set as is implied by the SM.

In FRACEP only the up-quark carries its own color-charge. The down-quark has color-charge only because it has an up-quark component. The proposed color-charge exchange process results when a down-quark base unit R_{d-} moves from an up-quark with one color to an up-quark with a different color. (In the SM both the up-quark and down-quark carry their own color-charge.)

Figure 2 shows the exchange process – the apparent exchange of color-charge between a green down-quark and a blue up-quark. The bond between the R_{d-} radical and its green up-quark component is broken (creating a green up-quark). With gluon interaction, the R_{d-} moves to a blue up-quark (to form a new blue down-quark) – completing the color exchange process.

6. Conclusions

We have shown that by allowing negative mass, a minimum set of only two fundamental particles (equal and opposite masses) can be used to construct composite versions of all of the SM particles and anti-particles. This model assumes that the fundamental particles have zero spin and charge; and, spin and charge are components that exhibit their properties as the result of dynamic behavior.

The multi-tiered fractal-like substructures allow a parallel pair of particle sets. One set, the “BU”, is the universe we see (the SM particles), and the other set, the “DU”, is the part of the universe we cannot see directly. The concept of dual universes has the possibility to offer some insight into the mystery of the dark matter that cosmologists believe populates three-fourths of the cosmos.

A number of unanswered questions have been swept under the rug by the simplicity of this heuristic model (structurally detailed as it may be). We have not addressed the issue of the inherent mass in FRACEP’s fundamental particles as opposed to gaining mass through a field exchange mechanism.

We have not addressed the bonding mechanism that maintains the substructures. Chemical models typically are based on electron exchange or sharing between atoms. A mechanism of this general type was assumed for FRACEP, but a possible exchange particle has not been considered.

It has been proposed that the stability and containment in composite structures might be modeled by the intertwined topological structures known as Borromean Rings (BR). Chichak, et al. [23] have exploited the BR mechanism in the context of the molecular bonding – supporting the construction of complex nanostructures. At the next smaller scale, Austin and Bertsch [24] have considered the

importance of BR in nuclei stability in the context of the presence or removal of halo neutrons in lithium isotopes.

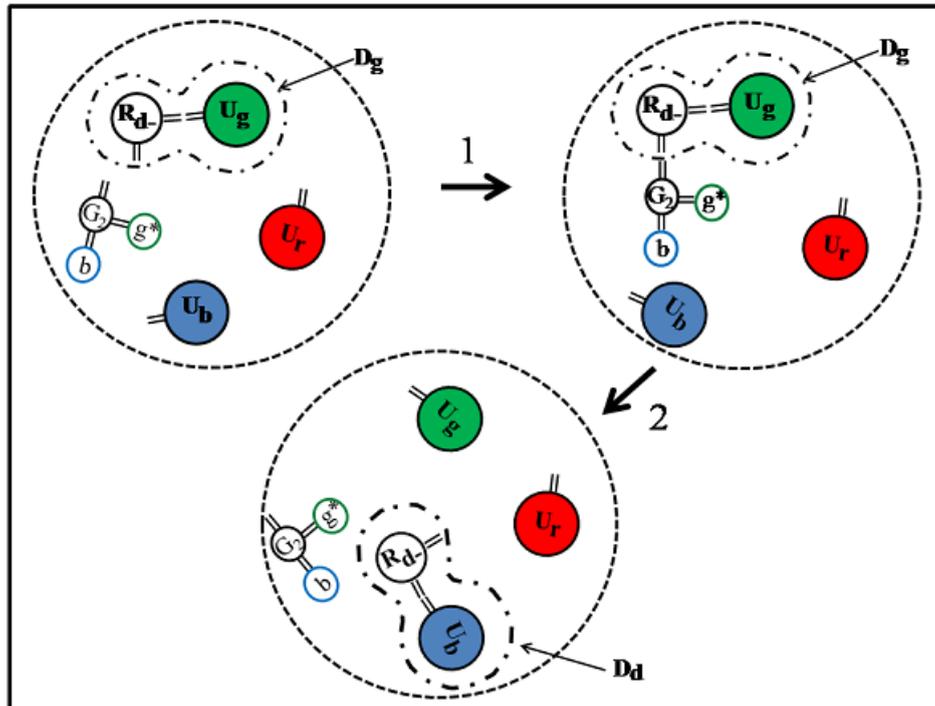


Figure 2. This shows a proposed color exchange process in a proton. The upper left proton contains a red up-quark, a blue up-quark and a green down-quark ($R_{d-} + U_g$). Within the proton is also a gluon with a blue color charge and a green anti-color charge. Step 1 shows the gluon rotating to attach to the R_{d-} (the upper right view). Step 2 shows the bond between R_{d-} and U_g broken leaving U_g as a free green up-quark. R_{d-} and U_b continue rotating and attach to form a blue down-quark (lower view). The gluon continues to rotate away from the newly formed quark.

At smaller scales yet, Chang [13] considers the possible relevance of BR to composite fractal-based particles. Their relevance to FRACEP’s substructures deserves consideration. One final option to explore is the possibility that gravitational effects alone (the oscillations in the FRACEP potential [15] at the smallest (fundamental) scales) explain the confinement of elements within the composite particles.

We assume dynamics account for the spin and charge effects, but a mechanism has not been established. Local geometry of rotating matter in the Einstein-Cartan theory has been suggested as possibly applicable to the spin mechanism [25 - 27].

Because of the composite nature of the fermions and bosons, the number of components (including charge carriers) grows as mass increases. This hints at the possibility of producing a total charge vs. total mass relationship and a mass spectrum predictive capability – a task yet to be completed.

Finally, the potential usefulness of solitons in modeling preons and quarks [3, 28] might be extended to the positive mass and negative mass components and fundamental particles in FRACEP as bright and dark soliton-like structures (similar to those used in nonlinear optics [29]). These issues all require further study.

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